ME 449 Robotic Manipulation
Spring 2014
Problem Set 3
Due Monday May 12 at beginning of class

Please note the correction to the course notes sent out by email on April 29. In numerical inverse kinematics, the "direction" to the desired configuration $X$ from $T$ (the end-effector configuration at the current guess for the joint angles), expressed in the $T$ frame, is given by

$$
[\mathcal{S}]=\log T^{-1} X
$$

1. Use Newton-Raphson iterative numerical root finding to perform two steps of finding the root of

$$
f(x, y)=\left[\begin{array}{l}
x^{2}-4 \\
y^{2}-9
\end{array}\right]
$$

when your initial guess is $\left(x_{1}, y_{1}\right)=(1,1)$. Write the general form of the gradient (for any guess $(x, y)$ ) and compute the results of the first two iterations. You can do this by hand or write a program. If you write a program, submit your code and the printout of the results. Also, give all of the correct roots, not just the one that would be found from your initial guess. How many are there?
2. Newton-Raphson for inverse kinematics for a 3 -DOF wrist: Exercise 1 from Chapter 6 (also see Figure 1 from class). For part (b), perform a single iteration of Newton-Raphson root finding using body-frame numerical inverse kinematics. First write the forward kinematics and Jacobian for general configurations of the wrist. Then apply your results for the specific case of an initial guess of zero joint angles, giving an end-effector frame at the identity, with a desired end-effector frame at

$$
X=R=\left[\begin{array}{rrr}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{array}\right] \in S O(3)
$$

If you write code, turn that in, in addition to the results.
3. Derive the dynamics of an RP arm using the Lagrangian formulation. The robot moves in gravity, the first link is treated as a point mass a distance $r_{1}$ from the first joint, and the second link is treated as a point mass a distance $\theta_{2}$ from the end of the first link, which is of length $L_{1}$ (see Figure 2 from class). After deriving the dynamics, write them in the standard form

$$
\tau=M(\theta) \ddot{\theta}+c(\theta, \dot{\theta})+g(\theta)
$$

Also give the Christoffel symbols for the mass matrix.


Figure 1: The 3-DOF wrist from class.


Figure 2: The RP arm from class.
4. Repeat the previous exercise using the Newton-Euler formulation (Chapters 8.3 and 8.4 ) with velocities expressed in the space frame.
5. You are given a rectangular block of uniform density $10 \mathrm{~kg} / \mathrm{m}^{3}$ and sides of length $10 \mathrm{~cm}, 20 \mathrm{~cm}$, and 30 cm . Write the inertia matrix of the body in a frame where $\hat{x}_{b}$ is aligned with the axis of minimum inertia and $\hat{z}_{b}$ is aligned with the axis of maximum inertia. You can either solve the volume integrals or consult any standard reference for inertia formulas. Explain your work.
6. For the body in the previous exercise, assume that the body frame is at $(1,0,0)$ in a world frame, with body axes aligned with the world axes. Assume the space velocity is given by $\mathcal{V}_{s}=\left(\omega_{s}, v_{s}\right)=(0,0,1,1,0,0)$.
(i) Write the body velocity $\mathcal{V}_{b}$.
(ii) Using concepts from Chapter 8.2, find the body force $\mathcal{F}_{b}$ needed to keep the body moving with a constant body velocity $\mathcal{V}_{b}$.

